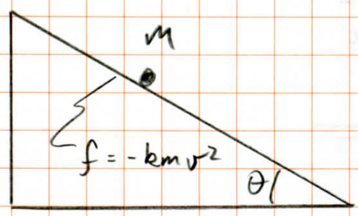
TMS PR 2.15

TMS 2-15 A PARTICLE SLIDES DOWN A PLANE. IF MOTION IS RESISTED BY $f = kmv^2$, SHOW THE TIME TO SLIDE d FROM REST IS

$$\underbrace{t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg\sin\theta}}}_{\checkmark}$$

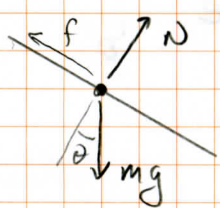


APPLY NSL

$$\Sigma F_{\parallel} = ma_{\parallel}$$

$$\cancel{m}g\sin\theta - k\cancel{m}v^2 = \cancel{m}v\frac{dv}{dx}$$

$$\int_0^x dx = \int_{v_0}^{v} \frac{v\,dv}{g\sin\theta - kv^2}$$



$$x = -\frac{1}{2k}\ln(g\sin\theta - kv^2\Big|_{v_0}^{v}$$

$$x = -\frac{1}{2k}\ln\left(\frac{g\sin\theta - kv^2}{g\sin\theta - kv_0^2}\right)$$

SOLVE FOR v:

$$\frac{g\sin\theta - kv^2}{g\sin\theta - kv_0^2} = e^{-2kx}$$

LET $v_0 \to 0 \Rightarrow v^2 = \frac{g}{k}\sin\theta\left(1 - e^{-2kx}\right)$

$$\frac{dx}{dt} = \sqrt{\frac{g}{k}\sin\theta}\sqrt{1 - e^{-2kx}}$$

$$\int_0^t dt = \int_0^x \frac{dx}{\sqrt{\frac{g}{k}\sin\theta}\sqrt{1-e^{-2kx}}} \quad \frac{\frac{1}{e^x}}{\frac{1}{e^x}}$$

$$t = \sqrt{\frac{k}{g\sin\theta}}\int \frac{e^{kx}}{\sqrt{e^{2x}-1}}\,dx$$

$\longrightarrow$

TMS Pr 2.15 CONTINUED

$$t = \sqrt{\frac{k}{g \sin\theta}} \int_0^x \frac{e^{kx}\,dx}{\sqrt{e^{2kx}-1}}$$

NOTING THAT (BLUE BOOK p 172)

$$\frac{d}{dx}\cosh^{-1}u = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

$$\Rightarrow \frac{d}{dx}\cosh^{-1}e^{kx} = \frac{ke^{kx}}{\sqrt{e^{2kx}-1}}$$

THE INTEGRAL IS

$$t = \sqrt{\frac{k}{g \sin\theta}}\,\frac{1}{k}\cosh^{-1}e^{kx}\Big|_0^d$$

$$\boxed{t = \frac{\cosh^{-1}e^{kd}}{gk\sin\theta}}$$

$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\cosh^{-1}(e^{\phi}) = \cosh^{-1}(1)$$

= ZERO

$$= \ln\left(1 + \sqrt{1-1}\right)$$

$$\cosh^{-1}(e^{\phi}) = \ln 1 = 0$$